Teams and Truthmakers

Søren Brinck Knudstorp and Robert van Rooij
ILLC and Philosophy, University of Amsterdam

Bochum, July 7, 2026

Directions in Relevant Logic

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Plan for the talk

- *Project:* Develop account of the meaning of OR, (AND, NOT,) MIGHT, MAY, MUST.
- *Two themes:*
  - New semantics for modals
  - Relevance, gluts, gaps, exactness
- *Structure:*
  1. Data and desiderata
     - Epistemic Contradictions
     - Free Choice
     - and more
  2. Our semantics
  3. Its explanations and predictions

Epistemic Contradictions

Consider:

(1) a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2) a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Epistemic Contradictions

Consider:

(1) a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2) a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Consider:

(1)
a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2)
a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Consider:

(1)
a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2)
a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Epistemic Contradictions

Consider:

(1)
a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2)
a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Epistemic Contradictions

Consider:

(1)
a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2)
a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Epistemic Contradictions

Consider:

(1)
a. # It is raining and possibly it is not raining $p \wedge \Diamond\neg p$
b. # It is raining and it might not be raining $p \wedge \Diamond\neg p$
c. # It is raining but it might not be raining $p \wedge \Diamond\neg p$
d. # It is not raining but it might be raining $\neg p \wedge \Diamond p$
e. # It might be raining but it is not raining $\Diamond p \wedge \neg p$
f. # It might not be raining but it is raining $\Diamond\neg p \wedge p$

Compare:

(2)
a. It is raining but it could have been snowing
b. It is here but it could have been there
c. I wrote this sentence but I didn't have to
d. You are reading this sentence but you don't have to

Task: Explain infelicity of $p \wedge \Diamond\neg p$ for epist. $\Diamond$.

Epistemic Contradictions: Gricean or not?

Compare Moorean sentences:

(3) a. # It is raining and I don’t know that it is raining
    b. # It is raining but I don’t know that it is raining

Gricean story supported by non-assertoric contexts:

a. Suppose it is raining and I don’t know that it is raining
b. If it is raining and I don’t know that it is raining, then . . .

In contrast:

(4) a. # Suppose it is raining and it might not be raining (Yalcin 2007)
    b. # If it is raining and it might not be raining, then . . .

Epistemic Contradictions: Gricean or not?

Compare Moorean sentences:

(3) a. # It is raining and I don’t know that it is raining
    b. # It is raining but I don’t know that it is raining

Gricean story supported by non-assertoric contexts:

a. Suppose it is raining and I don’t know that it is raining
b. If it is raining and I don’t know that it is raining, then . . .

In contrast:

(4) a. # Suppose it is raining and it might not be raining (Yalcin 2007)
    b. # If it is raining and it might not be raining, then . . .

Epistemic Contradictions: Gricean or not?

Compare Moorean sentences:

(3) a. # It is raining and I don't know that it is raining
    b. # It is raining but I don't know that it is raining

Gricean story supported by non-assertoric contexts:

a. Suppose it is raining and I don't know that it is raining
b. If it is raining and I don't know that it is raining, then ...

In contrast:

(4) a. # Suppose it is raining and it might not be raining (Yalcin 2007)
    b. # If it is raining and it might not be raining, then ...

Epistemic Contradictions: Gricean or not?

Compare Moorean sentences:

(3) a. # It is raining and I don't know that it is raining
b. # It is raining but I don't know that it is raining

Gricean story supported by non-assertoric contexts:

a. Suppose it is raining and I don't know that it is raining
b. If it is raining and I don't know that it is raining, then ...

In contrast:

(4) a. # Suppose it is raining and it might not be raining (Yalcin 2007)
b. # If it is raining and it might not be raining, then ...

Epistemic Contradictions: Gricean or not?

Compare Moorean sentences:

(3) a. # It is raining and I don't know that it is raining
    b. # It is raining but I don't know that it is raining

Gricean story supported by non-assertoric contexts:

a. Suppose it is raining and I don't know that it is raining
b. If it is raining and I don't know that it is raining, then . . .

In contrast:

(4) a. # Suppose it is raining and it might not be raining (Yalcin 2007)
    b. # If it is raining and it might not be raining, then . . .

Take-aways:

Take-aways:

(i) ECs to be explained by the meaning of epistemic modals

Take-aways:

(i) ECs to be explained by the meaning of epistemic modals

(ii) Non-trivial task since $p \wedge \Diamond\neg p$ is perfectly consistent on standard point-based relational accounts of modality

Free Choice (epistemic)

(5) a. Mr. X might be in Victoria or in Brixton (Zimmermann 2000)
b. $\rightsquigarrow$ Mr. X might be in Victoria

(6) a. Mr. X might be in Victoria or he might be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(7) a. Mr. X must be in Victoria or in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(8) a. Mr. X must be in Victoria or he must be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

Logical form:

- $\Diamond(p \vee q) \rightsquigarrow \Diamond p$
- $\Diamond p \vee \Diamond q \rightsquigarrow \Diamond p$
- $\Box(p \vee q) \rightsquigarrow \Diamond p$
- $\Box p \vee \Box q \rightsquigarrow \Diamond p$

Free Choice (epistemic)

(5) a. Mr. X might be in Victoria or in Brixton (Zimmermann 2000)
b. $\rightsquigarrow$ Mr. X might be in Victoria

(6) a. Mr. X might be in Victoria or he might be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(7) a. Mr. X must be in Victoria or in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(8) a. Mr. X must be in Victoria or he must be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

Logical form:

- $\Diamond(p \vee q) \rightsquigarrow \Diamond p$
- $\Diamond p \vee \Diamond q \rightsquigarrow \Diamond p$
- $\Box(p \vee q) \rightsquigarrow \Diamond p$
- $\Box p \vee \Box q \rightsquigarrow \Diamond p$

(5) a. Mr. X might be in Victoria or in Brixton (Zimmermann 2000)
b. $\rightsquigarrow$ Mr. X might be in Victoria

(6) a. Mr. X might be in Victoria or he might be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(7) a. Mr. X must be in Victoria or in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(8) a. Mr. X must be in Victoria or he must be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

Logical form:

- $\Diamond(p \vee q) \rightsquigarrow \Diamond p$
- $\Diamond p \vee \Diamond q \rightsquigarrow \Diamond p$
- $\Box(p \vee q) \rightsquigarrow \Diamond p$
- $\Box p \vee \Box q \rightsquigarrow \Diamond p$

Free Choice (epistemic)

(5) a. Mr. X might be in Victoria or in Brixton (Zimmermann 2000)
b. $\rightsquigarrow$ Mr. X might be in Victoria

(6) a. Mr. X might be in Victoria or he might be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(7) a. Mr. X must be in Victoria or in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(8) a. Mr. X must be in Victoria or he must be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

Logical form:

- $\Diamond(p \vee q) \rightsquigarrow \Diamond p$
- $\Diamond p \vee \Diamond q \rightsquigarrow \Diamond p$
- $\Box(p \vee q) \rightsquigarrow \Diamond p$
- $\Box p \vee \Box q \rightsquigarrow \Diamond p$

Free Choice (epistemic)

(5) a. Mr. X might be in Victoria or in Brixton (Zimmermann 2000)
b. $\rightsquigarrow$ Mr. X might be in Victoria

(6) a. Mr. X might be in Victoria or he might be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(7) a. Mr. X must be in Victoria or in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

(8) a. Mr. X must be in Victoria or he must be in Brixton
b. $\rightsquigarrow$ Mr. X might be in Victoria

Logical form:

- $\Diamond(p \vee q) \rightsquigarrow \Diamond p$
- $\Diamond p \vee \Diamond q \rightsquigarrow \Diamond p$
- $\Box(p \vee q) \rightsquigarrow \Diamond p$
- $\Box p \vee \Box q \rightsquigarrow \Diamond p$

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \lor q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \lor q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \lor \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \lor \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Free Choice (deontic)

(9) a. You may go to the beach or take the canal boat (Kamp 1973)
b. $\rightsquigarrow$ You may go to the beach

(10) a. You must go to the beach or the boat
b. $\rightsquigarrow$ You may go to the beach

(11) a. You may go to the beach or you may take the boat
b. $\rightsquigarrow$ You may go to the beach

But compare:

(12) a. You may go to the beach or you may take the boat, but I don't know which
b. $\not\rightsquigarrow$ You may go to the beach

- $\Diamond_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Box_d(p \vee q) \rightsquigarrow \Diamond_d p$
- $\Diamond_d p \vee \Diamond_d q \rightsquigarrow \Diamond_d p$ (usually)
- $\Box_d p \vee \Box_d q \rightsquigarrow \Diamond_d p$ (usually)

Paradox of Free Choice: $\Diamond p \overset{\text{mon.}}{\rightsquigarrow} \Diamond(p \vee q) \overset{\text{FC}}{\rightsquigarrow} \Diamond q$

Possibility inferences

Consider:

(13) a. Either Robert is following the talk online or he's busy with admin
b. $\rightsquigarrow$ Robert might be following the talk online
c. $p \vee q \rightsquigarrow \Diamond p$

Consider as well:

(14) ?? I have two or three children

Paradox of Possibility: $p \rightsquigarrow p \vee q \overset{\text{Pos.}}{\rightsquigarrow} \Diamond q$

Consider:

(13) a. Either Robert is following the talk online or he's busy with admin
b. $\rightsquigarrow$ Robert might be following the talk online
c. $p \vee q \rightsquigarrow \Diamond p$

Consider as well:

(14) ?? I have two or three children

Paradox of Possibility: $p \rightsquigarrow p \vee q \overset{\text{Pos.}}{\rightsquigarrow} \Diamond q$

Possibility inferences

Consider:

(13) a. Either Robert is following the talk online or he's busy with admin
b. $\rightsquigarrow$ Robert might be following the talk online
c. $p \vee q \rightsquigarrow \Diamond p$

Consider as well:

(14) ?? I have two or three children

Paradox of Possibility: $p \rightsquigarrow p \vee q \overset{\text{Pos.}}{\rightsquigarrow} \Diamond q$

Possibility inferences

Consider:

(13) a. Either Robert is following the talk online or he's busy with admin
b. $\rightsquigarrow$ Robert might be following the talk online
c. $p \vee q \rightsquigarrow \Diamond p$

Consider as well:

(14) ?? I have two or three children

Paradox of Possibility: $p \rightsquigarrow p \vee q \overset{\text{Pos.}}{\rightsquigarrow} \Diamond q$

Take-aways:

(i) Non-trivial to develop an adequate semantics;

Take-aways:
(i) Non-trivial to develop an adequate semantics;
(ii) TRUTH seems the wrong semantic notion for modal meaning.

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) THE ZIMMERMANN PROBLEM

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) THE EC EMBEDDING PROBLEM

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Zimmermann, Geurts, Aloni

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) THE ZIMMERMANN PROBLEM

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) THE EC EMBEDDING PROBLEM

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Zimmermann, Geurts, Aloni

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) THE ZIMMERMANN PROBLEM

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) THE EC EMBEDDING PROBLEM

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) THE ZIMMERMANN PROBLEM

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) THE EC EMBEDDING PROBLEM

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Zimmermann, Geurts, Aloni

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) ⇒ account for further phenomena,
(1) ⇒ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\leadsto$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Zimmermann, Geurts, Aloni

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\leadsto$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Starting point: Aloni's (2022) development of the Zimmermann–Geurts (2000, 2005) idea that possibility inferences are part and parcel of the meaning of disjunction.

Two departures from Aloni's system (called BSML):

(1) Different semantics for modals
(2) Relevant base rather than classical base

(2) $\Rightarrow$ account for further phenomena,
(1) $\Rightarrow$ resolve BSML modal issues, which include:

(15) The Zimmermann problem

a. It must be here or it must be there $\Box p \vee \Box q$
b. $\not\rightsquigarrow$ It must be here $\Box p$

(16) The EC Embedding problem

a. # Either it is raining but it might not be raining, or it is snowing but it might not be snowing. (Mandelkern 2019)

Desiderata

Desiderata

- Semantic account of EC and FC,

Desiderata

- Semantic account of EC and FC, while
  - avoiding overgeneration (Zimmermann problem),

Desiderata

- Semantic account of EC and FC, while
  - avoiding overgeneration (Zimmermann problem), and
  - predicting embedded ECs infelicitous

Desiderata

- Semantic account of EC and FC, while
  - avoiding overgeneration (Zimmermann problem), and
  - predicting embedded ECs infelicitous
- FC across modal flavours, but with
  - wide-scope deontics cancellable

Formal system

- (Information) states T are nonempty sets of ~~possible worlds~~ FDE-valuations, conceived as van Fraassenian (1969) truthmakers (or facts):

Definition

$$
\begin{aligned}
\mathrm{T}(p) &= \{\{\mathbf{p}\}\}, \\
\mathrm{T}(\neg\varphi) &= \mathrm{F}(\varphi), \\
\mathrm{T}(\varphi \wedge \psi) &= \mathrm{T}(\varphi) \otimes \mathrm{T}(\psi) = \{f \cup g \mid f \in \mathrm{T}(\varphi),\ g \in \mathrm{T}(\psi)\}, \\
\mathrm{T}(\varphi \vee \psi) &= \mathrm{T}(\varphi) \cup \mathrm{T}(\psi), \\
\\
\mathrm{F}(p) &= \{\{\overline{\mathbf{p}}\}\}, \\
\mathrm{F}(\neg\varphi) &= \mathrm{T}(\varphi), \\
\mathrm{F}(\varphi \wedge \psi) &= \mathrm{F}(\varphi) \cup \mathrm{F}(\psi), \\
\mathrm{F}(\varphi \vee \psi) &= \mathrm{F}(\varphi) \otimes \mathrm{F}(\psi).
\end{aligned}
$$

Formal system

- (Information) states T are nonempty sets of ~~possible worlds~~ FDE-valuations, conceived as van Fraassenian (1969) truthmakers (or facts):

Definition

$$
\begin{aligned}
\mathrm{T}(p) &= \{\{\mathbf{p}\}\},\\
\mathrm{T}(\neg\varphi) &= \mathrm{F}(\varphi),\\
\mathrm{T}(\varphi \wedge \psi) &= \mathrm{T}(\varphi) \otimes \mathrm{T}(\psi) = \{f \cup g \mid f \in \mathrm{T}(\varphi),\ g \in \mathrm{T}(\psi)\},\\
\mathrm{T}(\varphi \vee \psi) &= \mathrm{T}(\varphi) \cup \mathrm{T}(\psi),\\
\\
\mathrm{F}(p) &= \{\{\overline{\mathbf{p}}\}\},\\
\mathrm{F}(\neg\varphi) &= \mathrm{T}(\varphi),\\
\mathrm{F}(\varphi \wedge \psi) &= \mathrm{F}(\varphi) \cup \mathrm{F}(\psi),\\
\mathrm{F}(\varphi \vee \psi) &= \mathrm{F}(\varphi) \otimes \mathrm{F}(\psi).
\end{aligned}
$$

Formal system

- (Information) states T are nonempty sets of ~~possible worlds~~ FDE-valuations, conceived as van Fraassenian (1969) truthmakers (or facts):

Definition

$$\begin{aligned}
\mathrm{T}(p) &= \{\{\mathbf{p}\}\},\\
\mathrm{T}(\neg\varphi) &= \mathrm{F}(\varphi),\\
\mathrm{T}(\varphi \wedge \psi) &= \mathrm{T}(\varphi) \otimes \mathrm{T}(\psi) = \{f \cup g \mid f \in \mathrm{T}(\varphi),\ g \in \mathrm{T}(\psi)\},\\
\mathrm{T}(\varphi \vee \psi) &= \mathrm{T}(\varphi) \cup \mathrm{T}(\psi),\\
\\
\mathrm{F}(p) &= \{\{\overline{\mathbf{p}}\}\},\\
\mathrm{F}(\neg\varphi) &= \mathrm{T}(\varphi),\\
\mathrm{F}(\varphi \wedge \psi) &= \mathrm{F}(\varphi) \cup \mathrm{F}(\psi),\\
\mathrm{F}(\varphi \vee \psi) &= \mathrm{F}(\varphi) \otimes \mathrm{F}(\psi).
\end{aligned}$$

Formal system

- (Information) states T are nonempty sets of ~~possible worlds~~ FDE-valuations, conceived as van Fraassenian (1969) truthmakers (or facts):

Definition

$$\begin{aligned}
\mathrm{T}(p) &= \{\{\mathbf{p}\}\}, \\
\mathrm{T}(\neg\varphi) &= \mathrm{F}(\varphi), \\
\mathrm{T}(\varphi \wedge \psi) &= \mathrm{T}(\varphi) \otimes \mathrm{T}(\psi) = \{f \cup g \mid f \in \mathrm{T}(\varphi),\ g \in \mathrm{T}(\psi)\}, \\
\mathrm{T}(\varphi \vee \psi) &= \mathrm{T}(\varphi) \cup \mathrm{T}(\psi), \\
\\
\mathrm{F}(p) &= \{\{\overline{\mathbf{p}}\}\}, \\
\mathrm{F}(\neg\varphi) &= \mathrm{T}(\varphi), \\
\mathrm{F}(\varphi \wedge \psi) &= \mathrm{F}(\varphi) \cup \mathrm{F}(\psi), \\
\mathrm{F}(\varphi \vee \psi) &= \mathrm{F}(\varphi) \otimes \mathrm{F}(\psi).
\end{aligned}$$

Propositional clauses

Definition

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

$T \vDash p$ iff $\forall f \in T$: $\mathbf{p} \in f$

$T \dashv p$ iff $\forall f \in T$: $\overline{\mathbf{p}} \in f$

Then: $p \nvDash q \vee \neg q$, $p \wedge \neg p \nvDash q$, but also $p \nvDash p \vee q$

Propositional clauses

Definition

$T \vDash p$ iff $\forall f \in T$: $\mathbf{p} \in f$

$T \Dashv p$ iff $\forall f \in T$: $\overline{\mathbf{p}} \in f$

$T \vDash \neg\varphi$ iff $T \Dashv \varphi$

$T \Dashv \neg\varphi$ iff $T \vDash \varphi$

Then: $p \not\vDash q \vee \neg q$, $p \wedge \neg p \not\vDash q$, but also $p \not\vDash p \vee q$

Propositional clauses

Definition

$T \models p$	iff	$\forall f \in T$: $\mathbf{p} \in f$
$T \mathrel{=\!\!\!|} p$	iff	$\forall f \in T$: $\overline{\mathbf{p}} \in f$
$T \models \neg\varphi$	iff	$T \mathrel{=\!\!\!|} \varphi$
$T \mathrel{=\!\!\!|} \neg\varphi$	iff	$T \models \varphi$
$T \models \varphi \vee \psi$	iff	$\exists T', T''$ such that $T' \models \varphi; T'' \models \psi;$ and $T = T' \cup T''$

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

$$
\begin{array}{lll}
T \vDash p & \text{iff} & \forall f \in T\colon \mathbf{p} \in f \\
T \Dashv p & \text{iff} & \forall f \in T\colon \overline{\mathbf{p}} \in f \\
T \vDash \neg\varphi & \text{iff} & T \Dashv \varphi \\
T \Dashv \neg\varphi & \text{iff} & T \vDash \varphi \\
T \vDash \varphi \vee \psi & \text{iff} & \exists T', T'' \text{ such that } T' \vDash \varphi; T'' \vDash \psi; \text{ and } T = T' \cup T'' \\
T \Dashv \varphi \vee \psi & \text{iff} & T \Dashv \varphi \text{ and } T \Dashv \psi \\
T \vDash \varphi \wedge \psi & \text{iff} & T \vDash \varphi \text{ and } T \vDash \psi \\
T \Dashv \varphi \wedge \psi & \text{iff} & \exists T', T'' \text{ such that } T' \Dashv \varphi; T'' \Dashv \psi; \text{ and } T = T' \cup T''.
\end{array}
$$

Then: $p \not\vDash q \vee \neg q$, $p \wedge \neg p \not\vDash q$, but also $p \not\vDash p \vee q$

Propositional clauses

Definition

$$
\begin{array}{lll}
T \models p & \text{iff} & \forall f \in T\colon \mathbf{p} \in f \\
T \mathrel{=\!\!|} p & \text{iff} & \forall f \in T\colon \overline{\mathbf{p}} \in f \\
T \models \neg\varphi & \text{iff} & T \mathrel{=\!\!|} \varphi \\
T \mathrel{=\!\!|} \neg\varphi & \text{iff} & T \models \varphi \\
T \models \varphi \vee \psi & \text{iff} & \exists T', T'' \text{ such that } T' \models \varphi; T'' \models \psi; \text{ and } T = T' \cup T'' \\
T \mathrel{=\!\!|} \varphi \vee \psi & \text{iff} & T \mathrel{=\!\!|} \varphi \text{ and } T \mathrel{=\!\!|} \psi \\
T \models \varphi \wedge \psi & \text{iff} & T \models \varphi \text{ and } T \models \psi \\
T \mathrel{=\!\!|} \varphi \wedge \psi & \text{iff} & \exists T', T'' \text{ such that } T' \mathrel{=\!\!|} \varphi; T'' \mathrel{=\!\!|} \psi; \text{ and } T = T' \cup T''.
\end{array}
$$

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Propositional clauses

Definition

$$
\begin{array}{lll}
T \models p & \text{iff} & \forall f \in T\colon \mathbf{p} \in f \\
T \dashv p & \text{iff} & \forall f \in T\colon \overline{\mathbf{p}} \in f \\
T \models \neg\varphi & \text{iff} & T \dashv \varphi \\
T \dashv \neg\varphi & \text{iff} & T \models \varphi \\
T \models \varphi \vee \psi & \text{iff} & \exists T', T'' \text{ such that } T' \models \varphi; T'' \models \psi; \text{ and } T = T' \cup T'' \\
T \dashv \varphi \vee \psi & \text{iff} & T \dashv \varphi \text{ and } T \dashv \psi \\
T \models \varphi \wedge \psi & \text{iff} & T \models \varphi \text{ and } T \models \psi \\
T \dashv \varphi \wedge \psi & \text{iff} & \exists T', T'' \text{ such that } T' \dashv \varphi; T'' \dashv \psi; \text{ and } T = T' \cup T''.
\end{array}
$$

Then: $p \not\models q \vee \neg q$, $p \wedge \neg p \not\models q$, but also $p \not\models p \vee q$

Fact: for all φ, there is a greatest $T \vDash \varphi$; denote it $\mathrm{T}^*(\varphi)$.

Fact: for all φ, there is a greatest $T \vDash \varphi$; denote it $\mathrm{T}^*(\varphi)$.

Proposition: $\varphi \vDash_{\mathsf{FDE}} \psi$ if and only if $\mathrm{T}^*(\varphi) \vDash \psi$.

Two clauses for $\Diamond$.

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ (‘T rules φ in’); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

Let $\mathbf{Poss}(\alpha) := \{\beta \mid \alpha \vDash \Diamond\beta\}$.

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

Let $\mathbf{Poss}(\alpha) := \{\beta \mid \alpha \vDash \Diamond\beta\}$. Then:

Theorem (Double-barreled characterization)

$$\varphi \vDash \psi \qquad \text{iff} \qquad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{Poss}(\varphi) \supseteq \mathbf{Poss}(\psi).$$

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ (‘T rules φ in’); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt.

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ (‘T rules φ in’); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt. Compare the more implicit or covert way in which

- $T = \{\{\mathbf{p}\}, \{\overline{\mathbf{p}}\}\}$ does not rule out $p \wedge q$

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ (‘T rules φ in’); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt. Compare the more implicit or covert way in which

- $T = \{\{\mathbf{p}\}, \{\overline{\mathbf{p}}\}\}$ does not rule out $p \wedge q$

Thus we want a clause, $\Diamond_2$, on which:

- $T \vDash \Diamond_2(p \wedge q)$,

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt. Compare the more implicit or covert way in which

- $T = \{\{\mathbf{p}\}, \{\overline{\mathbf{p}}\}\}$ does not rule out $p \wedge q$

Thus we want a clause, $\Diamond_2$, on which:

- $T \vDash \Diamond_2(p \wedge q)$,
- $\{\{\mathbf{p}, \mathbf{q}\}, \{\mathbf{p}, \mathbf{r}\}\} \nvDash \Diamond_2 \neg p$,

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt. Compare the more implicit or covert way in which

- $T = \{\{\mathbf{p}\}, \{\overline{\mathbf{p}}\}\}$ does not rule out $p \wedge q$

Thus we want a clause, $\Diamond_2$, on which:

- $T \vDash \Diamond_2(p \wedge q)$,
- $\{\{\mathbf{p}, \mathbf{q}\}, \{\mathbf{p}, \mathbf{r}\}\} \nvDash \Diamond_2 \neg p$,
- $\{\{\mathbf{p}, \overline{\mathbf{p}}\}, \{\overline{\mathbf{p}}\}\} \vDash \Diamond_2(p \wedge \neg p)$.

Epistemic modality

Two clauses for $\Diamond$.

On one clause, we say that an information state T supports $\Diamond\varphi$ just in case part of T supports φ ('T rules φ in'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \vDash \varphi. \tag{1e}$$

(1e) is explicit or overt. Compare the more implicit or covert way in which

- $T = \{\{\mathbf{p}\}, \{\overline{\mathbf{p}}\}\}$ does not rule out $p \wedge q$

Thus we want a clause, $\Diamond_2$, on which:

- $T \vDash \Diamond_2(p \wedge q)$,
- $\{\{\mathbf{p}, \mathbf{q}\}, \{\mathbf{p}, \mathbf{r}\}\} \nvDash \Diamond_2\neg p$,
- $\{\{\mathbf{p}, \overline{\mathbf{p}}\}, \{\overline{\mathbf{p}}\}\} \vDash \Diamond_2(p \wedge \neg p)$.

Idea: we care whether φ is consistent *modulo*, or *relative to*, T'.

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

each $f \in T'$ consistently extends some $g \in T$.

- Say T supports $\lozenge_2\varphi$ if some refinement of T supports φ ('T does not rule out φ'); formally,

$$T \vDash \lozenge\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$\begin{aligned}
T &\vDash \Box\varphi & \text{iff} \qquad & T \vDash \varphi, \\
T &\dashv \lozenge\varphi & \text{iff} \qquad & T \dashv \varphi, \\
T &\dashv \Box_1\varphi & \text{iff} \qquad & \exists T \supseteq T' \dashv \varphi, \\
T &\dashv \Box_2\varphi & \text{iff} \qquad & \exists T \trianglerighteq T' \dashv \varphi.
\end{aligned}$$

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

$$\text{each } f \in T' \text{ consistently extends some } g \in T.$$

- Say T supports $\Diamond_2\varphi$ if some refinement of T supports φ ('T does not rule out φ'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$\begin{aligned} T &\vDash \Box\varphi && \text{iff} && T \vDash \varphi, \\ T &\Dashv \Diamond\varphi && \text{iff} && T \Dashv \varphi, \\ T &\Dashv \Box_1\varphi && \text{iff} && \exists T \supseteq T' \Dashv \varphi, \\ T &\Dashv \Box_2\varphi && \text{iff} && \exists T \trianglerighteq T' \Dashv \varphi. \end{aligned}$$

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

  each $f \in T'$ consistently extends some $g \in T$.

- Say T supports $\Diamond_2\varphi$ if some refinement of T supports φ (‘T does not rule out φ’); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$\begin{aligned}
T &\vDash \Box\varphi && \text{iff} && T \vDash \varphi,\\
T &\Dashv \Diamond\varphi && \text{iff} && T \Dashv \varphi,\\
T &\Dashv \Box_1\varphi && \text{iff} && \exists T \supseteq T' \Dashv \varphi,\\
T &\Dashv \Box_2\varphi && \text{iff} && \exists T \trianglerighteq T' \Dashv \varphi.
\end{aligned}$$

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

$$\text{each } f \in T' \text{ consistently extends some } g \in T.$$

- Say T supports $\Diamond_2\varphi$ if some refinement of T supports φ ('T does not rule out φ'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$\begin{aligned} T \vDash \Box\varphi &\qquad \text{iff} \qquad T \vDash \varphi, \\ T \mathrel{=\!\!|} \Diamond\varphi &\qquad \text{iff} \qquad T \mathrel{=\!\!|} \varphi, \\ T \mathrel{=\!\!|} \Box_1\varphi &\qquad \text{iff} \qquad \exists T \supseteq T' \mathrel{=\!\!|} \varphi, \\ T \mathrel{=\!\!|} \Box_2\varphi &\qquad \text{iff} \qquad \exists T \trianglerighteq T' \mathrel{=\!\!|} \varphi. \end{aligned}$$

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

each $f \in T'$ consistently extends some $g \in T$.

- Say T supports $\Diamond_2\varphi$ if some refinement of T supports φ ('T does not rule out φ'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$T \vDash \Box\varphi \qquad \text{iff} \qquad T \vDash \varphi,$$
$$T \Dashv \Diamond\varphi \qquad \text{iff} \qquad T \Dashv \varphi,$$
$$T \Dashv \Box_1\varphi \qquad \text{iff} \qquad \exists T \supseteq T' \Dashv \varphi,$$
$$T \Dashv \Box_2\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \Dashv \varphi.$$

Epistemic modality II

- Write $\mathsf{Inc}(f) = \{\mathbf{p} \mid \{\mathbf{p}, \overline{\mathbf{p}}\} \subseteq \mathbf{f}\}$
- Say f consistently extends g if $f \supseteq g$ and $\mathsf{Inc}(f) \subseteq \mathsf{Inc}(g)$
- Say T' refines T, written $T' \trianglelefteq T$, if

$$\text{each } f \in T' \text{ consistently extends some } g \in T.$$

- Say T supports $\Diamond_2\varphi$ if some refinement of T supports φ ('T does not rule out φ'); formally,

$$T \vDash \Diamond\varphi \qquad \text{iff} \qquad \exists T \trianglerighteq T' \vDash \varphi. \tag{2e}$$

- Let further:

$$\begin{aligned} T \vDash \Box\varphi &\qquad \text{iff} \qquad T \vDash \varphi, \\ T \dashv \Diamond\varphi &\qquad \text{iff} \qquad T \dashv \varphi, \\ T \dashv \Box_1\varphi &\qquad \text{iff} \qquad \exists T \supseteq T' \dashv \varphi, \\ T \dashv \Box_2\varphi &\qquad \text{iff} \qquad \exists T \trianglerighteq T' \dashv \varphi. \end{aligned}$$

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \models \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \models \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \models \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \models \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \models (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \models (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*

- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*

- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*
- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*

- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Predictions: EC

- Say that T is locally inconsistent if it contains an inconsistent fact
- φ is locally inconsistent if only locally inconsistent states support φ

Idea: Epistemic contradictions $p \wedge \Diamond\neg p$ are odd because they are locally inconsistent

- *The following are all locally inconsistent:*
  - $p \wedge \Diamond\neg p$, $\neg p \wedge \Diamond p$, $\Diamond p \wedge \neg p$, $\Diamond\neg p \wedge \neg p$
  - $(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q)$
  - $\Diamond(p \wedge \Diamond\neg p)$
  - $(p \wedge \Diamond\neg p) \vee \varphi$
- *We further avoid these BSML-inferences:*

- $\Diamond\varphi \wedge \Diamond\psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $\Diamond p \wedge \Diamond\neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash \Diamond(\varphi \wedge \Diamond\psi)$ (so also $p \vee \neg p \vDash \Diamond(p \wedge \Diamond\neg p)$)
- $\varphi \vee \psi \vDash (\varphi \wedge \Diamond\psi) \vee (\psi \wedge \Diamond\varphi)$ (so also $p \vee \neg p \vDash (p \wedge \Diamond\neg p) \vee (\neg p \wedge \Diamond p)$)

Epistemic contradictions are relevant:

$p \wedge \Diamond\neg p \models p$ but $p \wedge \Diamond\neg p \not\models q$

Epistemic contradictions are relevant:

$$p \wedge \Diamond\neg p \vDash p \quad \text{but} \quad p \wedge \Diamond\neg p \nvDash q; \text{ and}$$

$$\Box p \wedge \Diamond\neg p \vDash p \quad \text{but} \quad \Box p \wedge \Diamond\neg p \nvDash q.$$

Predictions: Epistemic FC

Valid FC and possibility inferences

$$\Diamond(\varphi \vee \psi) \vDash \Diamond\varphi \wedge \Diamond\psi,$$
$$\Diamond\varphi \vee \Diamond\psi \vDash \Diamond\varphi \wedge \Diamond\psi,$$
$$\varphi \vee \psi \vDash \Diamond\varphi \wedge \Diamond\psi,$$
$$\Box\varphi \vDash \Diamond\varphi,$$
$$\Box(\varphi \vee \psi) \vDash \Diamond\varphi \wedge \Diamond\psi,$$
$$\Box\varphi \vee \Box\psi \vDash \Diamond\varphi \wedge \Diamond\psi,$$
$$\neg\Diamond(\varphi \vee \psi) \vDash \neg\Diamond\varphi \wedge \neg\Diamond\psi.$$

Predictions: Epistemic FC

Valid FC and possibility inferences

$$\begin{aligned}
\Diamond(\varphi \vee \psi) &\vDash \Diamond\varphi \wedge \Diamond\psi, \\
\Diamond\varphi \vee \Diamond\psi &\vDash \Diamond\varphi \wedge \Diamond\psi, \\
\varphi \vee \psi &\vDash \Diamond\varphi \wedge \Diamond\psi, \\
\Box\varphi &\vDash \Diamond\varphi, \\
\Box(\varphi \vee \psi) &\vDash \Diamond\varphi \wedge \Diamond\psi, \\
\Box\varphi \vee \Box\psi &\vDash \Diamond\varphi \wedge \Diamond\psi, \\
\neg\Diamond(\varphi \vee \psi) &\vDash \neg\Diamond\varphi \wedge \neg\Diamond\psi.
\end{aligned}$$

Predictions: Zimmermannian puzzles

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18) a. $\varphi \vee \Box\psi \not\models \Box\psi$

b. Either these are not the winning numbers or I must have won the lottery

c. $\not\leadsto$ I must have won the lottery

(19) a. $\varphi \vee \Box\psi \not\models \psi$

b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18) a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\rightsquigarrow$ I must have won the lottery

(19) a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18) a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\leadsto$ I must have won the lottery

(19) a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Predictions: Zimmermannian puzzles

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18) a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\rightsquigarrow$ I must have won the lottery

(19) a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Predictions: Zimmermannian puzzles

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18) a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\rightsquigarrow$ I must have won the lottery

(19) a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Predictions: Zimmermannian puzzles

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18)
a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\rightsquigarrow$ I must have won the lottery

(19)
a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Predictions: Zimmermannian puzzles

Invalid inferences:

(17) $\Box\varphi \vee \Box\psi \not\models \Box\varphi \wedge \Box\psi$

(18)
a. $\varphi \vee \Box\psi \not\models \Box\psi$
b. Either these are not the winning numbers or I must have won the lottery
c. $\not\rightsquigarrow$ I must have won the lottery

(19)
a. $\varphi \vee \Box\psi \not\models \psi$
b. $\Box\varphi \vee \Box\psi \not\models \varphi \wedge \psi$

And

$$(\Box p \vee \Box q) \wedge \Diamond\neg q \wedge \Diamond\neg p$$

is **not** locally inconsistent.

Another Zimmermannian puzzle

	$\Box p \vee \Box\neg p$	it must be here or it must be there
$\rightsquigarrow$	$\Diamond p \vee \Diamond\neg p$	it might be here or it might be there
$\rightsquigarrow$	$\Diamond p \wedge \Diamond\neg p$	it might be here and it might be there
$\rightsquigarrow$	$\neg\Box\neg p \wedge \neg\Box p$	it is not that it must be there, and it is not that it must be here
$\rightsquigarrow$	$\neg(\Box\neg p \vee \Box p)$	it is not that it must be there or it must be here
$\rightsquigarrow$	$\neg(\Box p \vee \Box\neg p)$	it is not that it must be here or it must be there

Deontic modals

Deontic modals:

(i) The two possibility notions, $\Diamond_1, \Diamond_2$, remain

Deontic modals:

(i) The two possibility notions, $\Diamond_1, \Diamond_2$, remain

(ii) Modal flavour modelled via accessibility relation between states

Semantics for deontic $\Diamond_d$

Let R be relation between info states. Define:

$$T \models \Diamond_d \varphi \quad \text{iff} \quad \exists T\,R\,T' \models \Diamond\varphi,$$
$$T \models \Box\varphi \quad \text{iff} \quad \forall T\,R\,T' \models \Box\varphi \quad \text{iff} \quad \forall T\,R\,T' \models \varphi.$$

Deontic inferences

$\Diamond_d(\varphi \vee \psi) \models \Diamond_d\varphi \wedge \Diamond_d\psi,$ (always)

$\Diamond_d\varphi \vee \Diamond_d\psi \models \Diamond_d\varphi \wedge \Diamond_d\psi,$ (iff $(*)$ holds)

$\varphi \vee \psi \not\models \Diamond_d\varphi,$

$\Box_d(\varphi \vee \psi) \models \Diamond_d\varphi \wedge \Diamond_d\psi,$ (always)

$\Box_d\varphi \vee \Box_d\psi \models \Diamond_d\varphi \wedge \Diamond_d\psi,$ (iff $(*)$ holds)

$\Box_d\varphi \vee \Box_d\psi \not\models \Box_d\varphi,$

$\varphi \vee \Box_d\psi \not\models \Box_d\psi,$

$\neg\Diamond_d(\varphi \vee \psi) \models \neg\Diamond_d\varphi \wedge \neg\Diamond_d\psi.$

$(*) \quad \subseteq \circ R \quad \subseteq \quad R \circ \subseteq$

Semantics for deontic $\Diamond_d$

Let R be relation between info states. Define:

$$T \vDash \Diamond_d \varphi \quad \text{iff} \quad \exists T\, R\, T' \vDash \Diamond \varphi,$$
$$T \vDash \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \vDash \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \vDash \varphi.$$

Deontic inferences

$$\Diamond_d(\varphi \vee \psi) \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Diamond_d \varphi \vee \Diamond_d \psi \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\varphi \vee \psi \not\vDash \Diamond_d \varphi,$$
$$\Box_d(\varphi \vee \psi) \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Box_d \varphi \vee \Box_d \psi \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\Box_d \varphi \vee \Box_d \psi \not\vDash \Box_d \varphi,$$
$$\varphi \vee \Box_d \psi \not\vDash \Box_d \psi,$$
$$\neg \Diamond_d(\varphi \vee \psi) \vDash \neg \Diamond_d \varphi \wedge \neg \Diamond_d \psi.$$

$(*) \quad \subseteq \circ R \quad \subseteq \quad R \circ \subseteq$

Semantics for deontic $\Diamond_d$

Let R be relation between info states. Define:

$$T \models \Diamond_d \varphi \quad \text{iff} \quad \exists T\, R\, T' \models \Diamond \varphi,$$
$$T \models \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \models \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \models \varphi.$$

Deontic inferences

$$\Diamond_d(\varphi \vee \psi) \models \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Diamond_d \varphi \vee \Diamond_d \psi \models \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\varphi \vee \psi \not\models \Diamond_d \varphi,$$
$$\Box_d(\varphi \vee \psi) \models \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Box_d \varphi \vee \Box_d \psi \models \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\Box_d \varphi \vee \Box_d \psi \not\models \Box_d \varphi,$$
$$\varphi \vee \Box_d \psi \not\models \Box_d \psi,$$
$$\neg \Diamond_d(\varphi \vee \psi) \models \neg \Diamond_d \varphi \wedge \neg \Diamond_d \psi.$$

$(*) \quad \subseteq \circ R \quad \subseteq \quad R \circ \subseteq$

Semantics for deontic $\Diamond_d$

Let R be relation between info states. Define:

$$T \vDash \Diamond_d \varphi \quad \text{iff} \quad \exists T\, R\, T' \vDash \Diamond \varphi,$$
$$T \vDash \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \vDash \Box \varphi \quad \text{iff} \quad \forall T\, R\, T' \vDash \varphi.$$

Deontic inferences

$$\Diamond_d(\varphi \vee \psi) \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Diamond_d \varphi \vee \Diamond_d \psi \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\varphi \vee \psi \nvDash \Diamond_d \varphi,$$
$$\Box_d(\varphi \vee \psi) \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(always)}$$
$$\Box_d \varphi \vee \Box_d \psi \vDash \Diamond_d \varphi \wedge \Diamond_d \psi, \qquad \text{(iff } (*) \text{ holds)}$$
$$\Box_d \varphi \vee \Box_d \psi \nvDash \Box_d \varphi,$$
$$\varphi \vee \Box_d \psi \nvDash \Box_d \psi,$$
$$\neg \Diamond_d(\varphi \vee \psi) \vDash \neg \Diamond_d \varphi \wedge \neg \Diamond_d \psi.$$

$$(*) \quad \subseteq \circ R \quad \subseteq \quad R \circ \subseteq$$

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20)
a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21)
a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$

b. Mary might have gone by train. $\Diamond t$

c. Mary might have gone by bus. $\Diamond b$

d. Mary might not have gone by train. $\Diamond \neg t$

e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.

b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

More NL phenomena

1. Uncertainty inferences (via gaps, exactness, and $\Diamond_2$)

(20) a. Mary went by train or by bus. $t \vee b$
b. Mary might have gone by train. $\Diamond t$
c. Mary might have gone by bus. $\Diamond b$
d. Mary might not have gone by train. $\Diamond \neg t$
e. Mary might not have gone by bus. $\Diamond \neg b$

2. Indirectness of MUST (via gluts)
3. Failure of distributivity (via non-persistence)
4. Hurford disjunctions (via possibility inferences or exactness)

(21) a. # John studies in Denmark or in Copenhagen.
b. Mary went by train or by train and bus.

Thank you!